Enrollment No: ______________________ Exam Seat No:______________________

C. U. SHAH UNIVERSITY

Winter Examination-2020

Subject Name : Engineering Mathematics-I

Subject Code : 4TE01EMT2/4TE01EMT3 **Branch: B.Tech (All)**

Semester: 1 **Date: 09/03/2021** **Time: 03:00 To 06:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **[14]**

a) n^{th} derivative of $y = \frac{1}{x+a}$ is _____ **(01)**

a) $\frac{(-1)^n n!}{(x+a)^{n+1}}$ c) $\frac{(-1)^n n!}{(x+a)^n}$

b) $\frac{(-1)^{n-1} n!}{(x+a)^{n+1}}$ d) None of these

b) Let y_n denotes the n^{th} derivative of y, where $y = e^{-x}$ then $y_n =$___ **(01)**

a) $-e^{-x}$ c) $(-1)^{n+1} e^{-x}$

b) $(-1)^n e^{-x}$ d) None of these

c) The series $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$ represent expansion of ___ **(01)**

a) $\sin x$ c) $\sinh x$

b) $\cos x$ d) $\cosh x$

d) $1 + \frac{y^2}{2!} + \frac{y^4}{4!} + \frac{y^6}{6!}$ ... is the expansion of __ **(01)**

a) $\cos x$ c) $\sin x$

b) $\cos hx$ d) $\sin hx$

e) If $y = \cos^{-1} x$ then $x =$___ **(01)**

a) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-\cdots$ c) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}-\cdots$

b) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-\cdots$ d) None of these

f) Find $\lim_{x\to\infty} x^4 e^{-ax}$ **(01)**

a) 4 c) -4

b) 0 d) 1

g) Find theincorrect relation from following. **(01)**

a) $\sin ix = i\, sin\, hx$ c) $\tanh ix = i \tan x$

b) $\cos ix = -\cos hx$ d) $\sinh ix = i\, sin\, x$

h) If $u = y^x$ then $\frac{\partial u}{\partial x}$ is ___ **(01)**

a) xy^{x-1} c) $y^x \log x$

b) 0 d) None of these

i) If $x = r\cos\theta$ and $y = r\sin\theta$ then $\frac{\partial r}{\partial x}$ =__ **(01)**

a) $\sec\theta$ c) $cosec\,\theta$

b) $\sin\theta$ d) $\cos\theta$

j) If $= ax^2 + 2hxy + by^2$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ =___ **(01)**

a) $2u$ c) 0

b) u d) None of these

k) The conjugate of $z = 1 - 3i$ is____ **(01)**

a) $1-3i$ c) $3i$

b) $1+3i$ d) 1

l) Find $\det A$ where $A = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}$. **(01)**

m) Write an example of Upper Triangular Matrix. **(01)**

n) Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -8 & 0 & -1 \end{bmatrix}$ then find A^T. **(01)**

Attempt any four questions from Q-2 to Q-8 .

Q-2 **Attempt all questions** **[14]**

a) Let $y = \cos x \cos 2x \cos 3x$ then find y_n. **(05)**

b) If $y = (1-x^2)^{-\frac{1}{2}} \sin^{-1} x$, then show that $(1-x^2)y_{n+1} - (2n+1)xy_n - n^2 y_{n-1} = 0$ where y_n denotes n^{th} derivative of y. **(05)**

c) Find n^{th} derivative of $y = \log(ax+b)$. **(04)**

Q-3 **Attempt all questions** **[14]**

a) Expand $f(x) = \sec x$ in powers of x up to x^4 by Maclaurin's series. **(06)**

b) Using Taylor's series expansion ,expand $\log x$ in powers of $(x-2)$. **(05)**

c) If $y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ then show that $x = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \cdots$ **(03)**

Q-4 **Attempt all questions** **[14]**

a) Evaluate $\lim_{x\to 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$. **(05)**

b) Evaluate $\lim_{x\to\infty} \frac{1^2+2^2+3^2+\cdots+x^2}{x^3}$ **(05)**

c) Find y where $y = \lim_{x\to\frac{\pi}{4}} (1-\tan x)\sec 2x$ **(04)**

Q-5 **Attempt all questions** **[14]**

a) For $u = \tan^{-1}(\frac{y}{x})$,verify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ **(05)**

b) f is homogeneous function of degree 2 defined by **(05)**
$f(x,y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2+y^2}$ then show that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 2f(x,y) = 0$

c) Find $\frac{\partial^2 u}{\partial t^2}$ and $\frac{\partial^2 u}{\partial x^2}$,where $u = e^{x-at}\cos(x-at)$ .Is there any relation between $\frac{\partial^2 u}{\partial t^2}$ and $\frac{\partial^2 u}{\partial x^2}$? **(04)**

Q-6 **Attempt all questions** **[14]**

a) Using De-Moivre's theorem prove the following: **(06)**
1) $\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$
2) $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$

b) If $\sin(\alpha + i\beta) = x + iy$ then prove that $x^2\, cosec^2\alpha - y^2\sec^2\alpha = 1$. **(04)**

c) Find $\tanh x$ if $5\sinh x - \cosh x = 5$. **(04)**

Q-7 **Attempt all questions** **[14]**

a) Solve the system of linear equations: **(06)**

$$x + 2y = 3\,, y - z = 2\,, x + y + z = 1$$

b) Find the rank of $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 13 & 10 \end{bmatrix}$. **(05)**

c) Find eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. **(03)**

Q-8 **Attempt all questions** **[14]**

a) Using De Moivre's theorem expand $\sin^8\theta$ in a series of cosines of multiple of θ . **(06)**

b) For real value of z ,show that $\sinh^{-1} z = \log(z + \sqrt[2]{z^2+1})$ **(04)**

c) Simplify: $\frac{(\cos 3\theta + i\sin 3\theta)^{-2}(\cos 2\theta - i\sin 2\theta)^{\frac{3}{2}}}{(\cos 5\theta - i\sin 5\theta)^3(\cos 2\theta + i\sin 2\theta)^7}$ **(04)**